Polyhedral Model and MLIR

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Goals of this panel

Polyhedral model and integration inside MLIR?

1. What is the polyhedral model? (quick intro if needed)
2. Polyhedral model ecosystem:
    - What are the existing tools/compilers?
    - Current status of polyhedral compilation in MLIR?
3. What are the current/next steps? (Open discussion)

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These slides are only support/context for discussion

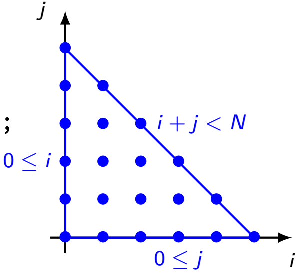
⇒ Feel free to interrupt/branch out on other topics

Polyhedral Model

- Exploit the regularity of structure/accesses of kernels

```
for (i=0; i<N; i++)
  for (j=0; j<N-i; j++)
S: A[i,j] = A[i,j-1] + B[j,i+2j];
```

$$\mathcal{D}_S = \{i, j \mid 0 \leq i, j \;\&\; i + j < N\}$$



- Concise mathematical representation of programs.
- Mathematical objects used to represent properties:
  - Set of integer points ⇒ $\mathcal{Z}$-polyhedron (Presburger set)
  - Relation between points ⇒ Affine function/relation
- Symbolic constants (parameters), usually for the problem size.

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used by

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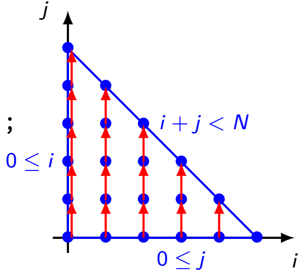
$$f_{dep} = (i,j \ \mapsto \ i,j+1)$$



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Why do we want polyhedral compilation?

- **Application domain:** Focus on affine computations
  - Linear algebra, stencil operations, dynamic programming, . . .

  ⇒ Machine learning, physical simulations, solver, . . .

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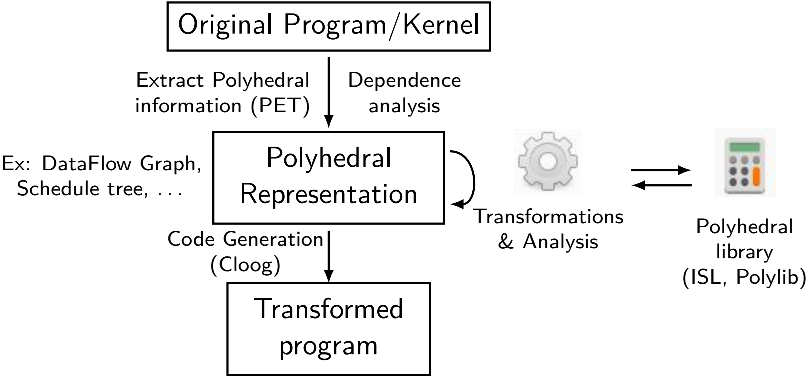
- (Non-exhaustive) **List of contributions:**
  - Many polyhedral loop transformation + static analysis:
    - Loop tiling, skewing, fusion, . . .
    - Automatic vectorization, parallelization, . . .
    - Dependence analysis, scheduling, cache modeling, . . .
    - Program verification, termination, . . .

Typical workflow of a polyhedral compiler



Original Program/Kernel

Extract Polyhedral information (PET)

Dependence analysis

Ex: DataFlow Graph, Schedule tree, . . .

Polyhedral Representation

Transformations & Analysis

Polyhedral library (ISL, Polylib)

Code Generation (Cloog)

Transformed program

Mathematical “toolbox” (library)

Library that provides representation/operations (ISL, Polylib)

- Basic operations:
  - Intersection, difference, emptiness check, . . .
  - Image, preimage, composition, . . .
- More complicated operations:
  - Finding a (parametric) lexicographic maximum (PipLib)
  - Counting integer points (Barvinok)
  - Transitive closure
- Careful with scalability ! (some algo have exponential cost)
  - It usually goes well, but not a guaranty (sometimes explode)

Resources about the polyhedral model (for newcomers)

- Website with bibliography list: `https://polyhedral.info/`
- IMPACT workshop (collocated with HiPeac conference)
- A few tutorial resources:
  - Tutorial by Sven Verdoolaege (barvinok library)
  - OpenScop: good introduction
  - Louis-Noël Pouchet: class material
  - ...
- Just grab one expert and start asking questions.

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- Useful tools for specific tasks:
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  - Benchmark suite: Polybench
  - And many more (lots of prototypes)...
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- Integration in mainstream compilers:
  - GCC ⇒ Graphite
  - LLVM ⇒ Polly

  Lessons on how successful (or not) they are/were?

MLIR dialects - `affine`

MLIR `affine` dialect: (+ `memref`)

- Good entry point for polyhedral analysis:
  - Express polyhedral program/fragments of program
  - Affine mapping manipulation (ex: `apply`)
  - `-lower-affine`: lower to `arith`/`scf`

- Affine dialect passes:
  - Some loop transformations (tiling, fusion, . . .)
  - Did not see any analysis (ex: dependence analysis)
  - ⇒ No elaborate analysis/heuristic/transformation.

Other pertinent parts of MLIR

Analysis/Presburger - "ISL/barvinok" in MLIR:

- `llvm-project/mlir/include/mlir/Analysis/Presburger`
- `IntegerRelation.h`: similar to ISL BasicMap/BasicSet
- `Barvinok.h` : Re-implem of Barvinok algorithm

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(Done? Maintained? How much from ISL is here?)

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MLIR `linalg` dialect:

- Tensor operations
- Can be lowered to `affine`, raising available from `affine`

Polygeist (Polymer)

Polygeist:

- Interfacing between MLIR and external polyhedral tools.
  - C code to `scf` then raise to `affine`.
  - `affine` Dialect ⇔ OpenScop
  - OpenScop: interface available for Pluto, Cloog, ISL

Discussions (if not already done)

- Any other efforts that was not listed?
  (Is there some unpublished efforts currently being made?)
- What form/shape should be the best for integrating polyhedral techniques in this ecosystem?
- What are the next steps?
- Further coordination/discussion